Reply to 'comment on Berry phase in a composite system'

X. X. Yi[1], L. C. Wang [1], and T. Y. Zheng [2]
[1] *Department of Physics, Dalian University of Technology, Dalian 116024, China*
[2] *School of Physics, Northeast Normal University, Changchun 130024,China*

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In their comment[1] on our Letter[2], Han and Guo point out that we did not obtain quantitatively correct results for the Berry phase in the composite system, and the proposed subsystem Berry phase is not well-defined. Then they present a calculation for the Berry phase in the weak coupling limit $g \to 0$ and discuss the definition of the subsystem geometric phase. While the eigenstate $|\Psi_{1,3}(\phi,\theta,g)\rangle$ in the limit $g \to 0$ may be correct in their comment, the Berry phase is not the result as Han *etal.* presented in their comment for a realistic composite system. As we will show, the contribution of the second subsystem(the subsystem does not feel the magnetic fields) to the Berry phase in the composite system tends to zero in the limit $g \to 0$ in the system, and the geometric phase for the subsystem is well defined.

We now turn to the details. The composite system is degenerate at $g = 0$. Thus we need to study the non-Abelian geometric phase at this point. Take the eigenvalue E_1 as an example, the two degenerate orthogonal eigenstates at $g = 0$ are[3], $|\Psi_{a,b}\rangle = (\cos\frac{\theta}{2}e^{-i\phi}|\uparrow\rangle + \sin\frac{\theta}{2}|\downarrow\rangle)_1 \otimes \frac{1}{\sqrt{2}}(|\downarrow\rangle \pm e^{i\phi}|\uparrow\rangle)_2$. Define $A_{xy} = i\langle\Psi_x|\frac{\partial}{\partial\phi}|\Psi_y\rangle$, $x, y = a, b$, we obtain

$$A = \begin{pmatrix} \cos^2\frac{\theta}{2} - \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \cos^2\frac{\theta}{2} - \frac{1}{2} \end{pmatrix}. \tag{1}$$

As the non-Abelian geometric phase is not a gauge invariant quantity, one can not measure all the matrix elements of $U_g = \mathcal{P}exp(i\oint A d\phi)$, except the eigenvalues of U_g and its trace, which in this case contain no contribution from the second subsystem. It is important to note that $|\Psi_a^{'}\rangle = (\cos\frac{\theta}{2}e^{-i\phi}|\uparrow\rangle + \sin\frac{\theta}{2}|\downarrow\rangle)_1 \otimes |\uparrow\rangle_2$ and $|\Psi_b^{'}\rangle = (\cos\frac{\theta}{2}e^{-i\phi}|\uparrow\rangle + \sin\frac{\theta}{2}|\downarrow\rangle)_1 \otimes |\downarrow\rangle_2$ are also (orthogonal degenerate) eigenstates of the composite system at $g = 0$, which also lead to zero contribution of the second subsystem to the Berry phase in the composite system. On the other hand, the adiabatic condition in the limit $g \to 0$ is $\sim |\omega/(g\sin\theta)| \ll 1$ for time-independent θ and g, where $\omega = \partial\phi/\partial t$. This tells that *the adiabatic theorem would break down in the weak coupling limit.* A very small chosen ω leads the system to difficulties to finish a cyclic evolution due to decoherence effects.

Collecting the idea in [4, 5, 6], we now explain that the definition of subsystem geometric phase given in our paper is well defined. To make it clear, we first map the mixed state of the subsystem into the Bloch ball, then prolong the density matrix vector until it reaches the unitary Bloch sphere. The two vectors that represent the two eigenstates of the density matrix point in opposite directions, respectively. The weighted sum of the individual PHASES is then understood as the weighted sum of the AREAS corresponding to the two pure states lying on the unitary Bloch sphere. As the individual phase is specified as the area and the two vectors are a jointed pair, the defined subsystem geometric phase is gauge invariant and well defined.

In conclusion, we show that in the weak coupling limit $g \to 0$, the second subsystem makes no contribution to the geometric phase in realistic composite systems, instead of $(-\pi)$ in the comment. As we have emphasized in [7], the definition $\gamma = \sum_i p_i\gamma_i$ may act as weighted one-particle geometric phase, it is $U(1)$ gauge invariant as shown in [5] and well defined as we have explained.

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[3] They come from [1], where the authors take them as the eigenstates of the composite system in the limit $g \to 0$. Note that ϕ together with θ characterize the direction of the magnetic field which drives only the first subsystem.
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